Mathematics Department

"A computer is the mathematician's best friend"

μ-Games

January 2025

Rules:

The idea of this event is to gap the bridge between mathematics and programming. When working on these exercises, we hope that the participant will have a better understanding of the underlying mathematical concepts. You will not be required to do many difficult programming tasks. With array manipulation and basic functionality, you should be able to solve all the exercises.

When working on these exercises, you must follow the following rules.

- You are allowed to work in groups of a maximum of 4 people.
- You will have 3 hours to solve the problems.
- For problems, you can use the default maths library of your programming language (for example *import math* in Python). Other libraries like *numpy* cannot be used.
- You cannot look up any computer code that may help you solve the problem.

After 3 hours, the solutions to the exercises will be discussed. To check your own solution, go to the website http://clover.science.uu.nl/dj.

A standard setup of your code could look like this, where *function()* is the method used to solve the problem.

```
# First, we read the size of the testset (if necessary).
n = input()
arr = []

for _ in range(n):
    row = input().split()
    arr.append(row)

def function():
    pass

answer = function()

# Finally, we print the answer of the testset.
print(answer)
```

Problem 1: Interesting Integral

Difficulty: ★ ☆ ☆ ☆ ☆
Keywords: Calculus

The end is coming in the final of the ADSF (Annual Dutch Snowball Fight), and finalist Iris wonders if she has enough snowballs left to outlive her opponent Ido. She looks at her stock, and to her own surprise notices that her stock of snowballs resembles the function $x^n e^x$, between some real values a, b. Of course, if she can know the value of the integral, she can estimate the number of snowballs (in L) she has left. Thus, your task is to determine the value of this integral

$$I(a, b, n) = \int_a^b x^n e^x \, \mathrm{d}x,$$

for specified values of a, b, n.

Input

- One line with float $0 \leq a \leq 1000$.
- One line with float $0 \leq b \leq 1000$.
- One line with integer $1 \leq n \leq 1000$.

Output

- Output the value of the integral $I(a, b, n)$ with precision 10^{-6}.

Examples

Input	Output
0 1 2	0.718282

Problem 2: Morning Madness

Difficulty: ★ ★ ☆ ☆ ☆
Keywords: Combinatorics

Mike the milipede always has trouble getting ready in the morning and therefore is always late for his work as an ice sculptor. One day, his boss, Mia, the mamba, has enough. "Why does it take you so long each day to get here?" She asks. Mike explains that even when he wakes up early it just takes too long to put on his socks and shoes. "There are just so many possible orders in which I can put them on!" he says. Mia of course does not relate at all, as she is a mamba. "Well, how many orders are there?" Mia asks impatiently. Mike is flabbergasted by the question, as he does not know.

Mike has n feet and puts a sock and a shoe on each of them to prepare. Of course, Mike must put on a sock before putting on a shoe on that same foot, as otherwise he would look like a fool! Can you help Mike figure out the number M of possible orders to get dressed each morning?

Input

- An integer $1 \leq n \leq 100$, denoting the number of feet Mike has.

Output

- Output M, the possible orders for Mike to put on his socks and shoes, modulo $10^9 + 7$.

Examples

Input	Output
2	6
4	2520

Problem 3: RSA problem

Difficulty: ★ ★ ★ ☆ ☆
Keywords: Number theory, Cryptography

In the RSA crypto system, the private key consists of a pair of primes p, q. The public key is then given by $N = pq$ and some $e \in \mathbb{N}$, such that e is co-prime with $(p-1)(q-1)$. We can now encrypt a message m by calculating $c = m^e \mod N$. We can decrypt this cyphertext using the private key by first calculating $d = e^{-1} \mod (p-1)(q-1)$ and then calculating $m = c^d \mod N$.

Alice wants to give a present to her friends Bob and Charlie. To prepare, she asks her friends to secretly send her their favourite numbers encrypted using the RSA crypto scheme. Alice creates the public keys using her favourite prime and a randomly generated prime.

Given as input are two combinations of public keys and the messages sent to Alice. Your task is to find Bob's and Charlie's favourite numbers. The exponent has the value $e = 65537$.

Input

- Two lines. On each line a public key, exponent and the message sent. (N, c)

Output

- The sum of Bob and Charlie's favourite numbers.

Examples

Input	Output
1948570357 825297009 2415141767 1175818504	363

Problem 4: Infinite Chessboard

Difficulty: ★ ★ ★ ☆ ☆
Keywords: Discrete Mathematics, Chess

One of the best things to do on a cold winter night is to play a nice game of chess. Of course, since there are only 64 squares, this can be quite boring. Therefore, one of the organisers of the μ-games invited infinite chess, which is played on an infinite chess board. In infinite chess, there are a lot of different tactics, one of which involves only the knight. This knight-flee™ tactic basically says that it could be advantageous to move the knight as far away from the battle as possible, only to later strike into the battle for victory. Assume that the knight starts on $(0, 0)$.

Part a: Minimum Moves

Determine the minimum number of moves required to move the knight to $(x, 0)$, given an integer $x \in \mathbb{Z}$.

Input

- A single integer $0 < x \leq 10000$.

Output

- A single integer representing the minimum number of moves.

Examples

Input	Output
5	3

Input	Output
8	4

Part b: Counting Paths

Determine how many distinct paths of minimum length exist for the knight to travel from $(0, 0)$ to $(x, 0)$.

Input

- A single integer $0 < x \leq 10000$.

Output

- A single integer representing the number of distinct paths of minimum length. (Consider using modular arithmetic if the number is very large.)

Examples

Input	Output
1	6

Input	Output
12	20

Problem 5: Frozen Fish

Difficulty: ★ ★ ★ ★ ☆
Keywords: Algebra, Number Theory

On a cold winter day on the south pole, the penguin Chrystal looks in her refrigerator for some frozen fish to prepare dinner. To her surprise, she finds out that her refrigerator is completely empty! Quickly, she puts on her warm coat and leaves to buy some fish at the market.

In the market, there are two fish shops. Alice's shop Antarctic Anchovy only sells fish in packs of a fish. The other store, Brisk Bass, owned by Bob, only sells fish in packs of b fish.

To cook dinner, Chrystal needs c fish. She wonders whether it is possible or not to buy exactly c fish from Alice's and Bob's stores. Of course, in the first case, she would like to know how many packs she needs to buy in Alice's store and how many packs she needs from Bob. Can you help Chrystal?

Input

- One line containing two space separated integers $1 \leq a, b \leq 100$, denoting the sizes.
- One line containing $c < 10^6$, the number of fish Chrystal wants to buy.

Output

- Output two space separated integers $x, y \geq 0$ such that $ax + by = c$. If there are multiple solutions, output the solution (x, y) with minimal x. If it is not possible to buy c fish by buying packs of a and b fish, output 'Impossible!'

Examples

Input	Output
3 1 3	0 3

Input	Output
10 32 2025	Impossible!